Teaching the Truth Table for the Material Conditional: Overcoming the Stipulation Problem

Lars Enden and Noah Prentice

Abstract

As every teacher of introductory logic can attest, it is a significant challenge to get students to accept the truth table for the material conditional. The main sticking point is that students can be reluctant to accept that a material conditional is true whenever its antecedent is false. In this paper, we examine some popular strategies for teaching the truth table for the material conditional. Ultimately, we find them all lacking because they all rely—in one way or another—on stipulating that the truth table is correct. We argue that stipulation is a less than ideal way to teach the truth table for the conditional, and we, therefore, recommend a new strategy that actually *shows* students why the material conditional is true when its antecedent is false.

Keywords: Material Conditional, Truth Table, Logic, Pedagogy of Logic

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Every teacher of introductory deductive logic needs a strategy for tackling the truth table for the material conditional, as shown in Figure 1.

Figure 1

The Truth Table for the Material Conditional

P	Q	$P \supset Q$
T	T	T
T	F	F
F	T	T
F	F	T

The main trouble, of course, is those pesky last two lines where the antecedent of the conditional is false. Students are reluctant—and with good reason—to accept that the conditional is true in those two cases. So, every teacher of logic needs some strategy for explaining to students why we put Ts on those two lines. But what is a *good* strategy for doing this? In this paper, we examine some popular methods of teaching the truth table for the material conditional, and we argue that they are unsatisfactory because they depend, some more blatantly than others, on merely *telling* the students that the truth table is correct without really explaining *why* it is correct. We have, therefore, developed a new strategy that avoids such stipulation by actually *showing* the students why the material conditional is true when its antecedent is false.

What is Wrong with Stipulation?

We first argue that stipulation is an undesirable way to teach the truth table for the material conditional. Consider, for example, the following excerpt from Tim Button (2018).[1]

> I'm just going to come clean and admit it: conditionals are a big old mess in [Truth-Functional Logic]. Exactly how much of a mess they are is a matter of philosophical contention. [...] For now, I am going to stipulate the following: $P \supset Q$ is false if and only if P is true and Q is false (Button and Magnus 2018, 33, symbols modified for clarity).

Let us call this approach, "the direct stipulation strategy." The main advantage of this strategy is that it is quick and does not get the students hung up on the philosophical debate surrounding conditionals. However, we do not think that the direct stipulation strategy is an acceptable way to teach the truth table for the material conditional. While we agree that conditionals are, as Button says, "a big old mess," and we accept that stipulation has its place in logic pedagogy, we think that the kind of stipulation involved in this strategy is not pedagogically acceptable because it lacks explanatory force. In other words, we believe that whenever stipulation is used in the teaching of the truth tables, it should be used in such a way that the students can appreciate *why* the stipulation is being made.

By way of contrast, let us consider a kind of stipulation often used in the teaching of truth tables that we believe *is* pedagogically acceptable. When teaching the truth table for disjunction, instructors will often point out the difference between inclusive and exclusive uses of the word "or" in ordinary language. Then instructors will often stipulate that the truth table for disjunction will follow the inclusive usage. Students may initially balk at this stipulation, but once the instructor says something like, "Don't worry, we can capture the meaning of the exclusive sense by using this new operator together with our other operators," students are generally contented to allow it. We suggest that this is a pedagogically acceptable—or good—kind of stipulation because it is possible to explain to students in a straightforward manner why the stipulation is being made. We further suggest that a stipulation is not pedagogically acceptable—or bad—if it fails to have the same kind of explanatory force. That is, a bad stipulation occurs when an instructor is either unable or unwilling to explain *why* such a stipulation should be accepted by the students, and a good kind of stipulation occurs when the instructor adds some explanation for why the stipulation should be accepted. The direct stipulation strategy is the bad variety because the instructor can offer nothing better than, "It's complicated, so just trust me." Such a stipulation lacks explanatory force, and, therefore, also lacks pedagogical force. We suggest that such stipulations should be avoided as much as possible when teaching the truth tables.

Therefore, we would do well to look for a strategy for teaching the truth table for the material conditional that does not involve stipulation, but, as we will argue, all of the popular strategies involve implicit stipulations of the bad variety. Ultimately, then, they have the same problem—although less obviously—as the direct stipulation strategy.

The Lying Strategy

Another common strategy for justifying the truth table for the material conditional is what we call the lying strategy. The main idea behind this strategy is for the instructor to present the students with an example of a conditional statement in the future tense and then ask whether the speaker of the sentence lied in cases where the antecedent turns out false. The following excerpt is Patrick J. Hurley and Lori Watson's (2018) treatment.

> [I]magine that your logic instructor made the following statement: "If you get an A on the final exam, then you will get an A for the course." Under what conditions would you say that your instructor had lied to you? [...] [W]hat if you failed to get an A on the final exam? Two alternatives are then possible: Either you got an A for the course anyway (false antecedent, true consequent) or you did not get an A for the course (false antecedent, false consequent). In neither case, though, would you say that your instructor had lied to you. Giving her the benefit of the doubt, you would say that she had told the truth" (Hurley and Watson 2018, 344).

Hurley and Watson seem to be suggesting that since the instructor only *clearly* lies in the case when the student gets an A on the final exam but does not get an A for the class, "giving her the benefit of the doubt" dictates that she told the truth in all other cases.

A variation of this approach is presented by Brooke Noel Moore and Richard Parker (2021), who rely on the concept of a broken promise rather than on the concept of lying. Their example is "If Moore's paycheck arrives this morning, then Moore will buy lunch" (Moore and Parker 2021, 308). Clearly, this promise will be broken if Moore's paycheck arrives this morning, but he does not buy lunch. Moore and Parker claim that there are no other circumstances where Moore will have broken his promise.

> And *that* is why the truth table has a conditional false in one and only one case, namely, where the antecedent is true and the consequent is false. (In everyday communication, when we discover the antecedent of a conditional is false, we normally just forget about the claim. Here, we say it's true because it clearly isn't false.)" (308).

Just like Hurley and Watson's strategy, Moore and Parker's strategy relies on the idea that we are to give the benefit of the doubt to the maker of a conditional promise. If the antecedent of the conditional promise turns out false, we should say that the promise is kept since it is not *clearly* broken.

The lying strategy constitutes at least two improvements over the direct stipulation strategy. First, and most obviously, it gives *some* kind of justification for the truth table. Secondly, it reinforces the idea that our logical operators are supposed to be tracking ordinary natural language to some degree. However, the bad kind of stipulation is still at work in this strategy.

Todd Furman (2008) rightly criticizes the lying strategy, noting that the idea of "benefit of the doubt" does not and should not play any role in justifying the truth tables: "The truth values of compound statements should be independent of our attitudes towards the speaker—they should be a function of the truth value of their constituent simple statements and the logical operators" (Furman 2008, 180). We agree with this assessment. The notion of "benefit of the doubt" is doing a lot of heavy lifting in this strategy, and when we look at it carefully, we find that it is actually a subtle kind of stipulation. The direct stipulation strategy asks the students simply to concede that a material conditional is true whenever its antecedent is false, and the lying strategy adds only that this concession should be made based on a principle of charity that requires giving people the benefit of the doubt before accusing them of lying or promise breaking. Despite its advantages over the direct stipulation strategy, the lying strategy still involves the bad kind of stipulation.

One might object that the concept of lying or promise breaking is being used in this context in an attitude-independent manner to mean something like "the telling of a falsehood." But, if so, then asking "Under what conditions would you say that so-and-so lied to you?" would mean the same as "Under what conditions is so-and-so's statement false?" And this is the very question we are trying to answer. So, an analysis based on an attitude-independent concept of lying would do no explanatory work, and, therefore, the strategy once again relies on the bad kind of stipulation. Given this reliance of the lying strategy on stipulation—a reliance that does not depend on lying's attitudinal content—we ought to look for alternatives.

The Logical Equivalence Strategy

We now turn to what we think is the most promising strategy for teaching the truth table for the material conditional: the logical equivalence strategy. The general approach of this strategy is to argue that the truth table for the material conditional must be equivalent to some other well-formed formula, whose truth table is independently justified. Furman (2008), for example, argues that the truth table for the material conditional must be what it is because $P \supset Q$ is logically equivalent to $\sim P \vee Q$; so, these two propositions must have exactly the same truth table. Similarly, Irving M. Copi, Carl Cohen, and Kenneth McMahon (2016) point out that $P \supset Q$ is logically equivalent to $\sim(P \cdot \sim Q)$; so they must have exactly the same truth table.

Because we cannot prove the purported logical equivalences until *after* we have convinced the students to accept the truth tables, the main pedagogical force of these strategies lies in how intuitive the students are likely to find these logical equivalences. We have come across two different methods for getting students intuitively to accept the logical equivalence. One approach takes a syntactic angle by trying to establish a rule for transforming conditional sentences into sentences with different main operators. The other approach takes a semantic angle by trying to build up the truth table from a consideration of the common features of conditional statements in ordinary discourse. We begin with the the syntactic approach.

The Syntactic Approach

Todd Furman's (2008) proposed method grounds the needed logical equivalence in the students' intuitions of natural language and logical relationships. His idea is to give an example of a statement in natural language which can be formulated both as an implication and as a disjunction. He then concludes that if the same sentence can be formulated two different ways, then the two formulations must be equivalent. Therefore, the two formulations must have the same truth table.

The strategy begins by choosing a good sample sentence. The one Furman uses is "Take the antivenin, or die," which is formalized as the disjunction $P \vee Q$, where P is the statement "you take the antivenin" and Q is the statement "you will die."[2] Also, Furman explains, this sentence clearly means the same thing as "If you don't take the antivenin, then you will die," which is formalized as the material conditional $\sim P \supset Q$. So, we arrive at a general rule for translating disjunctions into conditionals: "A disjunction may be converted into an equivalent conditional as long as the first disjunct of the disjunction changes its sign (is negated) and the logical operator '$\vee$' is change[d] to a '$\supset$' " (Furman 2008, 181). Then, without any explanation, Furman claims, "the reverse procedure converts a conditional into a logically equivalent disjunction" (181). This gives us one version of the syntactic rule of replacement often called material implication; that is, it establishes the logical equivalence between $P \supset Q$ and $\sim P \vee Q$. Using this equivalence, along with the truth tables for disjunction and negation, we get the truth table for the material conditional. This result is shown in figure 2.

Figure 2

The Logical Equivalence Between Disjunction and Material Conditional

P	Q	$\sim P \vee Q$	$P \supset Q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Like the lying strategy, the syntactic approach to the logical equivalence strategy has the advantage of connecting logic back to natural language. It also has the advantage of giving the students an impression of how syntactic rules are intuitively justified by natural language, which is something they will almost certainly encounter again later in the course.

Unfortunately, Furman's strategy depends on the bad kind of stipulation.[3] The main problem is that the disjunction involved in the example is the exclusive disjunction rather than the inclusive disjunction. This is important because if the disjunction is read as exclusive, then the procedure does not show the needed logical equivalence. In other words, if we read "Take the antivenin, or die" to mean "Either you will take the antivenin, or you will die, *but not both*," then it is logically equivalent to "You will not take the antivenin *if and only if* you will die." The question is: upon hearing the sentence "Take the antivenin, or die," will the students understand it as suggesting that taking the antivenin and dying is a potential outcome or not. To us, "you will die" more likely means something like "you will die *from the venom*" and not "you will die *sometime in the future*," which gives the disjunction an exclusive meaning. To test the alternative, we could change the "or" to the more awkward, but less ambiguous, "and/or". So, let us consider "You will take the antivenin and/or you will die." Now it is clear that we are not claiming that taking the antivenin will necessarily save you: you may die anyway. We now ask, is it as intuitively obvious that this is logically equivalent to "If you don't take the antivenin, you will die?" We don't think so. It is certainly harder to see than Furman's presentation makes it seem, but we need something like this if we are to avoid stipulation in the procedure.

We think that Furman's example has intuitive appeal mostly because it is likely to be understood by the students as a pairing of a biconditional with an exclusive disjunction rather than a pairing of a conditional with an inclusive disjunction. In other words, we think that the students are likely to understand "Take the antivenin, or die" to mean $P \veebar Q$, where $\veebar$ is the exclusive disjunction, which is logically equivalent to $\sim P \equiv Q$, not $\sim P \supset Q$. For the procedure to avoid this stipulation, it requires a clear example of an inclusive (not exclusive) disjunction, and its associated material conditional (not material equivalence). This turns out to be a very hard thing to manufacture. Let us try to "reverse engineer" such an example by taking a conditional that is unlikely to be understood as a biconditional. One possibility might be a sentence like "If you do not study, then you will not pass the exam."[4] But even this example would be easy to interpret as a biconditional. Will students tend to interpret this to mean that if they study they definitely *will* pass the exam? If so, then this example does not quite fit the bill. Instead, let us try something that they almost certainly will not interpret as a biconditional. Something like the old standard "If it is raining, then the streets are wet." This is logically equivalent to "Either it is not raining, or the streets are wet." The disjunction in this case is clearly inclusive, but is the logical equivalence between these two sentences intuitively obvious? We think not. This goes some way toward supporting our contention that it is either impossible or extremely difficult to manufacture a clear example of a material conditional and its associated inclusive disjunction for which the logical equivalence between them is intuitively obvious. Without such an example, the syntactic approach to the logical equivalence strategy does not fully succeed.

One might object here[5] that Furman's procedure does not have to establish logical equivalence to succeed. If we can intuitively establish that $\sim P \vee Q$ logically implies $P \supset Q$ rather than that the two are logically equivalent, that may be enough. Once this logical implication is established, the argument would run like this: $\sim P \vee Q$ is true whenever P is false, and since $\sim P \vee Q$ implies $P \supset Q$, $P \supset Q$ is also true whenever P is false; therefore, a material conditional is true whenever its antecedent is false. The problem with this argument is that it also relies on the disjunction being understood as inclusive rather than exclusive. The claim that $\sim P \vee Q$ is true whenever P is false only works for an inclusive reading of the disjunction; it does not work under an exclusive reading. This is because $\sim P \veebar Q$ is false when P is false and Q is true. Therefore, even if this shortened procedure were acceptable, we would still have to establish intuitively that a clearly inclusive disjunction implies its associated material conditional, and this has the same problems as trying to establish their logical equivalency.

We admit that it is incredibly unlikely that a student in an introductory logic course would be able to spot these difficulties. However, that does not mean that the issues are not serious. Most logic instructors will encounter a large number of students, so the likelihood of an instructor encountering such an eagle-eyed and quick-witted student *at some point* in their career might not be as unlikely as one might think. Additionally, giving logic students inadequate reasons to accept the truth table for the conditional is clearly in opposition to the values and principles of the course, whether or not the students are likely to notice. Therefore, in order for Furman's logical equivalence strategy to be entirely successful, an instructor must be able to establish the logical equivalence between the material conditional and its related disjunction based on intuition alone, and we think that this task is a near impossibility unless the student already has a clear understanding of the truth table for the material conditional in the first place. Therefore, the syntactic approach to the logical equivalence strategy seems to require the bad kind of stipulation.

The Semantic Approach

The semantic approach, most notably expressed by Copi, Cohen, and McMahon (2016), is a variation on the logical equivalence strategy. Unlike Furman's syntactic approach, which involves the syntactic rule that relates the material conditional to its corresponding disjunction, the semantic approach involves building the truth table for the material conditional by appealing to common meanings among different uses of conditionals in ordinary language. However, just like Furman's syntactic approach, the semantic approach of Copi, Cohen, and McMahon also relies on the bad kind of stipulation. Therefore, it is also not a completely successful strategy.

Copi, Cohen, and McMahon use their strategy to teach the truth tables for both the disjunction and the material conditional. In both cases, there are four steps to the procedure: (1) emphasize the differences between different senses of the operator in ordinary language; (2) create a truth table that represents the common partial meaning shared between these senses; (3) show that the common partial meaning is adequate for retaining the validity of a distinctive argument form that uses that operator; and (4) identify the operator's truth table to that of the common partial meaning (Copi, Cohen, and McMahon 2016, 320).[6]

This strategy is appealing for at least three reasons. First, it establishes a connection to natural language, similar to the lying strategy and to Furman's syntactic approach. Second, it emphasizes the importance of considering the different uses of words and our ability to represent these different meanings using logical symbols. Third, it explicitly endorses the idea that the formal representation of these logical operators should "play nicely" with the other operators. After all, if the formalization of a disjunction were to invalidate the argument forms which we care about, that would clearly indicate an error in the formalization process.

Despite the advantages, though, this strategy ultimately fails because it contains a hidden stipulation, which, in the case of the disjunction, is the good kind of stipulation, but, in the case of the material conditional, is the bad kind of stipulation. To see this, let us look at how Copi, Cohen, and McMahon apply the strategy to the disjunction. Step 1: they note that "or" is used in ordinary language in at least two senses, the inclusive sense and the exclusive sense. They give examples of each of these senses. Step 2: they suggest that the meaning shared between these two different sense—what they call the "common partial meaning"—is that at least one of the two disjuncts is true, which is exactly what the inclusive sense of "or" says. In other words, the common partial meaning happens to line up perfectly with the inclusive sense of "or." Step 3: they show that the inclusive sense of "or" is sufficient to retain the deductive validity of disjunctive syllogism, which they take to be the distinctive logical argument form for "or." Step 4: they conclude that the truth table for $\vee$ is adequate to capture the meaning of "or" for logical purposes.

The strategy applies to the material conditional is a similar way. Step 1: they give several stock examples of different types of conditional claims in ordinary language. Step 2: they claim, "No matter what type of implication is asserted by a conditional statement, part of its meaning is the negation of the conjunction of its antecedent with the negation of its consequent" (Copi, Cohen, and McMahon 2016) Step 3: they note that treating $P \supset Q$ as logically equivalent to $\sim(P \cdot \sim Q)$ is adequate for retaining the deductive validity of distinctive logical argument forms involving the conditional, like modus ponens and modus tollens. Step 4: they conclude that $P \supset Q$ is logically equivalent to $\sim(P \cdot \sim Q)$, and therefore they have the same truth table (324).

We can now see where stipulations come into these applications of the strategy. It comes in at step 2. Copi, Cohen, and McMahon move from a *partial* meaning to a *completed* truth table. How do they do this? If we are considering the common *partial* meaning shared by all uses of an operator, then we should expect that the result would be a *partial* truth table, not a completed one. Copi, Cohen, and McMahon are essentially stipulating that any line of the common partial truth table where two different ordinary uses of the operator differ ought just to default to T. Let us return to step 2 for the disjunction for a moment to see this more clearly. They claim that the inclusive sense of "or" captures the common partial meaning shared between both the inclusive and exclusive senses of "or," but this essentially privileges a truth-value of T over a truth-value of F. In contrast, we believe that the real representation of the common partial meaning of the disjunction is not really the inclusive sense of "or;" it is actually the partial truth table shown in Figure 3.

Figure 3

The Common Partial Truth Table for the Disjunction

P	Q	$P \vee Q$
T	T	?
T	F	T
F	T	T
F	F	F

This partial truth table represents the common partial meaning shared between the inclusive and exclusive senses of "or" since it leaves open the areas of disagreement. When Copi, Cohen, and McMahon suggest that the common partial meaning is captured by the inclusive sense, they are merely stipulating that a T rather than an F should be put in place of the ?. But why should that be? It does no good to point out that the validity of disjunctive syllogism is retained under the inclusive sense of "or" because the same can be said of the exclusive sense of "or." In other words, we can fill in the ? with either T or F and disjunctive syllogism will still be a valid argument form. But, as we have already mentioned, stipulating which sense of "or" we will adopt as our operator is the good kind of stipulation since it is easy to explain why this stipulation is acceptable, namely because both uses of "or" can be defined in terms of one another once we have our full complement of operators. However, when we look at the common partial meaning strategy as applied to the $\supset$, the stipulation goes from good to bad.

Just like our treatment of the common partial meaning of $\vee$, we suggest that the common partial meaning of the conditional is expressed by the truth table in Figure 4.

Figure 4

The Common Partial Truth Table for the Conditional

P	Q	$P \supset Q$
T	T	T
T	F	F
F	T	?
F	F	?

Once again Copi, Cohen, and McMahon suggest that the ?s should just default to Ts. Here is how they explain it.

> The previous proposal to translate both inclusive and exclusive disjunctions by means of the symbol $\vee$ was justified on the grounds that the validity of the disjunctive syllogism was preserved even if the additional meaning that attaches to the exclusive "or" was ignored. Our present proposal to translate all conditional statements into the merely material conditional symbolized by $\supset$ may be justified in exactly the same way. Many arguments contain conditional statements of various kinds, but the validity of all valid arguments of the general type with which we will be concerned is preserved even if the additional meanings of their conditional statements are ignored (Copi, Cohen, and McMahon 2016, 324).

Apparently, Copi, Cohen, and McMahon believe that putting an F in one of the ?s in the common partial truth table adds "additional meaning," but this is stipulation. One could just as well say that a ? should default to F, and that replacing a ? with a T creates "additional meaning." Indeed, it seems as though neither of the resulting truth tables would have *more* meaning than the other; they would merely have *different* meanings. Therefore, the claim that replacing ?s with Fs adds "additional meaning" is a stipulation, and it is the bad kind because there is no explanation for why this stipulation should be accepted. We are simply told, without explanation, to accept that Fs add additional meaning and that Ts do not. Furthermore, it does no good to rely on the fact that the validity of modus ponens and modus tollens is preserved by the material conditional since, just like with disjunctive syllogism, modus ponens and modus tollens will still be valid no matter what truth values are put in for the ?s in the common partial truth table for $\supset$.[7]

Therefore, the common partial meaning approach as presented in Copi, Cohen, and McMahon (2016) turns out to rely on the bad kind of stipulation. Still, we think that their strategy is very promising, and we borrow heavily from it in our own strategy, which teaches the truth table for the conditional in a way that respects ordinary language usage but at the same time makes a convincing case, without relying on mere stipulation, that a conditional is true whenever its antecedent is false. We turn to that strategy now.

A New Approach

We suggest a new approach to teaching the truth table for the material conditional. Our approach is a logical equivalence strategy since we will rely on a certain logical equivalence involving the material conditional. However, it is designed to avoid stipulation; we want to *show* the students why the material conditional is true when its antecedent is false. Still, we are borrowing fairly heavily from other logical equivalence strategies. From the syntactic approach of Furman, we borrow the idea of using examples from natural language to support the intuition that the needed logical equivalence holds, and from the semantic approach of Copi, Cohen, and McMahon we borrow the idea of starting with a common partial truth table and logically working out what should replace the missing truth values.

Before our technique can be applied, the students should already have been taught the other truth tables. Most importantly, they should be familiar with the truth tables for conjunction and for the material biconditional. We will also take it for granted that they have no problem with the second line of the truth table for the material conditional.[8] These required truth tables are collected together in Figure 5.

Figure 5

The Truth Tables Needed for the Procedure

P	Q	$P \cdot Q$	$P \supset Q$	$P \equiv Q$
T	T	T	?	T
T	F	F	F	F
F	T	F	?	F
F	F	F	?	T

This setup requires that the truth table for $\equiv$ be taught before the truth table for $\supset$, which is the opposite of how many, if not most, logic teachers proceed. It is common to rely on the truth table for the conditional to justify the truth table for the biconditional. Hurley and Watson, for example, note that the truth table for the biconditional is "required by the fact that $P \equiv Q$ is simply a shorter way of writing $(P \supset Q) \cdot (Q \supset P)$" (Hurley and Watson 2018, 345). Such a justification will not be possible for our strategy. The truth table for the biconditional will have to be justified independently of the truth table for the conditional, but this can be done intuitively. Just like with the other truth tables, we suggest sticking to examples from natural language to do this. A few examples are usually all that is needed to elicit the intuition that $P \equiv Q$ means that P and Q have the same truth value in all cases. Such classic examples as "Joe is a bachelor if and only if Joe is an unmarried man" or "3 is an even number if and only if 3 is divisible by 2" are good candidates for this treatment. The important thing is that the truth table for the material biconditional must be justified independently of the truth table for the material conditional. Our experience shows that this approach does not usually cause any problems for the students; they are generally willing to accept the truth table for the biconditional without needing support from a pre-established truth table for the conditional.

Only *after* establishing the truth table for the biconditional do we bring in the definition of $P \equiv Q$ as $(P \supset Q) \cdot (Q \supset P)$. Because we want to avoid stipulation and because we have established the truth table for the biconditional independently of the truth table for the conditional, this definition must be intuitively justified. We suggest returning to the intuitive examples: remind the students that "Joe is a bachelor if and only if Joe is an unmarried man" means that "Joe is a bachelor" and "Joe is an unmarried man" have the same truth value. Then point out that another way to say this is to say that *if* "Joe is a bachelor" is true, *then* "Joe is an unmarried man" is also true *and if* "Joe is an unmarried man" is true, *then* "Joe is a bachelor" is also true." Using Examples like this can intuitively establish the result that $P \equiv Q$ and $(P \supset Q) \cdot (Q \supset P)$ are logically equivalent without simply stating that this is so. An instructor who has not tried this strategy before might worry that the students are going to struggle with this equivalence: how can they understand the meaning of $(P \supset Q) \cdot (Q \supset P)$ if they do not already understand the truth table for the material conditional? This is a fair question, but our experience suggests that the students only need an intuitive understanding of conditional talk at this point. The students do not need to fully grasp the truth table for the material conditional to understand that "P and Q have the same truth value" can be stated conditionally as "if P is true, then Q is true, and if Q is true, then P is true."[9] The students will generally have a fair sense of what this means even though they are not in a good position to answer awkward questions about conditionals such as "But what about when the antecedents are false?"[10]

If students are willing to accept this much, then we are now in position to show them why the rest of the lines of the truth table for $\supset$ are T. We take the truth tables we have so far and replace $P \equiv Q$ with the logically equivalent statement $(P \supset Q) \cdot (Q \supset P)$, as shown in Figure 6.

Figure 6

Replacing the Biconditional

P	Q	$P \cdot Q$	$P \supset Q$	$(P \supset Q)$	$\cdot$	$(Q \supset P)$
T	T	T	?		T	
T	F	F	F		F	
F	T	F	?		F	
F	F	F	?		T	

We now use the truth function for conjunction to fill in some details of this truth table. We point out that a conjunction is true when (and only when) both conjuncts are true. Since the biconditional is true on both the top and bottom rows, and, since the definition of the biconditional turns out to be a conjunction, both of these conjuncts must be true on the top and bottom rows. The result is shown in Figure 7.

Figure 7

Working Out the Truth Table

P	Q	$P \cdot Q$	$P \supset Q$	$(P \supset Q)$	$\cdot$	$(Q \supset P)$
T	T	T	?	T	T	T
T	F	F	F	?	F	?
F	T	F	?	?	F	?
F	F	F	?	T	T	T

But the left-hand conjunct of the definition of $P \equiv Q$ just *is* $P \supset Q$. Therefore, the first and last line of the truth table for $P \supset Q$ must be T. Figure 8 shows this result.

Figure 8

Two Lines Determined

P	Q	$P \cdot Q$	$P \supset Q$	$(P \supset Q)$	$\cdot$	$(Q \supset P)$
T	T	T	**T**	T	T	T
T	F	F	F	?	F	?
F	T	F	?	?	F	?
F	F	F	**T**	T	T	T

This gets us most of the way to our goal. Unfortunately, no amount of working out the rest of this truth table is going to help us fill in that third line. So, from this point onward, we no longer need the definition of $P \equiv Q$ or the truth table for conjunction. So, let us get rid of the truth table for conjunction, and put $P \equiv Q$ back on our table. Figure 9 shows this new set up.

Figure 9

The Intermediate Partial Truth Table for $\supset$

P	Q	$P \supset Q$	$P \equiv Q$
T	T	T	T
T	F	F	F
F	T	?	F
F	F	T	T

Now what about that third line? Well, if it were F, then our truth table for $P \supset Q$ would be exactly the same as for $P \equiv Q$. But we know from our intuitive examples that this is not correct. We have shown that $P \equiv Q$ is logically equivalent to $(P \supset Q) \cdot (Q \supset P)$. If $P \supset Q$ and $P \equiv Q$ had the same truth table, then we could have shortened our work considerably by merely defining $P \equiv Q$ as $P \supset Q$. If the students need more convincing of this fact, we suggest more examples from ordinary discourse: "if there is a fire in this room, then there is oxygen in this room" clearly (and thankfully) does not mean the same thing as "there is a fire in this room if and only if there is oxygen in this room". Therefore, the third line of the truth table for $P \supset Q$ must be T as shown in Figure 10.

Figure 10

The Final Truth Table for $\supset$

P	Q	$P \supset Q$	$P \equiv Q$
T	T	T	T
T	F	F	F
F	T	**T**	F
F	F	T	T

This completes the procedure.

The last maneuver in our procedure requires further comment because there clearly is *some* stipulation there.[11] We are essentially saying to the students, "The third row must be either T or F; and if it were F, then the conditional and the biconditional would be logically equivalent, which they clearly are not. Therefore, the third row must be T." The stipulation at work here is that, on that third row, $P \supset Q$ must be either T or F, which assumes a bivalent logic. In fact, it has been suggested to us[12] that our procedure may be overly complicated for no good reason. If we are going to assume a bivalent logic, then we should use a much simpler "eliminative" argument along the following lines. Suppose that it has been established that the first two lines of the truth table for $P \supset Q$ are T and F respectively, then—assuming bivalence—there are only four possibilities for the other two lines. These four possibilities are shown in Figure 11.

Figure 11

Potential Truth Tables for $\supset$

P	Q	$P \supset_1 Q$	$P \supset_2 Q$	$P \supset_3 Q$	$P \supset_4 Q$
T	T	T	T	T	T
T	F	F	F	F	F
F	T	**T**	**T**	**F**	**F**
F	F	**T**	**F**	**T**	**F**

The only one of these possible truth tables that is plausible for "if... then..." is $\supset_1$. The other three possibilities would make the conditional logically equivalent either with its own consequent ($\supset_2$), with the biconditional ($\supset_3$), or with a conjunction ($\supset_4$). None of these is plausible. Therefore, if we are assuming bivalence, then we must pick one of these four to finish out the truth table, and the only one that makes clear sense is $\supset_1$.

We suggest that there are at least three things that recommend our procedure over the eliminativist procedure outlined above. First, we agree with Joseph S. Fulda (1989) that the eliminativist procedure is not very effective at showing students *why* a material conditional with a false antecedent is true. Our procedure is designed to give an explanation to this effect. We can at least say to the students, "the material conditional is true when its antecedent is false because the definition of the biconditional—a definition that you all accepted—would be nonsense otherwise." Second, our procedure is less abstract than the eliminativist procedure. When we are at the stage of explaining the basic truth functions, we think it is best to stay within the realm of natural language and intuition as much as possible. Some students may simply not be ready to tackle abstract arguments concerning possible truth functions before the basic ones are fully grasped. Third, waiting to assume bivalence until after the last line of the truth table is established takes some of the sting out of worries about truth value gaps. Suppose, for example, that a student is not impressed with our procedure and complains that we have assumed that the third line must be T or F rather than "undetermined" or "not applicable" or some other such alternative. Because the procedure has already established that the last line is T, an instructor can offer something along the following lines: "we have already established that the material conditional is true in *some* cases where the antecedent is false, and if this much is established, there seems to be very little reason to continue to be uneasy about the other case." Although this is far from a proof, it may go some way toward alleviating the unease that some students feel in the face of the stipulation of bivalence. And at least now the stipulation comes with an explanation, so it is the good kind of stipulation.

Another potential worry about our procedure is that it is a long way to go to establish the truth table for the material conditional and that, while other versions of the logical equivalence strategy may not be perfect, they get there much more quickly. To this objection, we respond that the journey is worth it for two important reasons. First, as we have noted several times, it avoids the bad kind of stipulation, which we believe is antithetical to the entire project of teaching symbolic logic. Second, it gives students a little preview of what is to come in their logic studies by showing them how we can sometimes prove surprising things using truth table techniques. These reasons are more than enough to justify taking the time to walk students through our procedure.

Needless to say, justifying the truth table for the material conditional to students is hard. We have looked at several popular strategies, and we have argued that all of them rely on the bad kind of stipulation in one way or another. Such stipulation provides no justification for the truth table for the material conditional, but asks that students to accept it merely on trust. Involving the concepts of lying or promise breaking seems to help, but the attitudinal content of these concepts make them less than desirable in the context of formal logic. A strategy based on logical equivalence has the best chance of success, but neither Furman nor Copi, Cohen, and McMahon presented a strategy that did not ultimately fall back on stipulation. As a result, their approaches do only slightly better than the direct stipulation strategy. We, therefore, recommend our strategy, which gives students a strong, reason to accept the truth table for the material conditional without relying on the bad kind of stipulation to do so.

Notes

1. Nothing like this excerpt appears in the predecessor of this book, *forallx* (Magnus 2017), Therefore, we attribute the authorship of this passage solely to Button.

2. Actually, Furman says that $P =$ "take the antivenin" and $Q =$ "die," but he clearly does not mean this literally since these are commands rather than propositions with truth values.

3. There is another stipulation in Furman's procedure that it worth mentioning, although we think that it can be avoided. The stipulation lies in the fact that Furman's logical equivalence is not quite the one we need. Because we are trying to explain the truth table for the $\supset$, we want to establish the logical equivalence between $P \supset Q$ and $\sim P \vee Q$ rather than the logical equivalence between $\sim P \supset Q$ and $P \vee Q$. The negation sign is in a different location from Furman's example. So Furman's equivalence is not exactly the one we need. Because Furman offers no explanation for why his equivalence suffices, his procedure involves the stipulation that one of these logical equivalences is just as good as the other. However, this stipulation is easily remedied by changing the example to something of the right form. Perhaps "If you take the poison, you will die" would suffice. Therefore, we think that this stipulation is not very serious, and we will let it pass by without further comment.

4. An example suggested by an anonymous referee

5. This interesting objection was pointed out by an anonymous referee

6. The treatment in Copi, Cohen, and McMahon (2016) only lists the first three steps, but the fourth step is clearly implied, so we have added it here for clarity.

7. There is a slight complication here because validity is usually defined conditionally as "if the premises are true, then the conclusion must be true." So, we cannot exactly determine the validity of an argument until we have settled on the meanings of conditional claims, which is exactly what is at issue. However, we will leave aside this worry for now.

8. As an anonymous referee pointed out, some students are uncomfortable with the first line of the truth table for the material conditional on the grounds that, even when both the antecedent and consequent are true, the material conditional does not intuitively seem true when the two are not conceptually related to one another. For example, some student might not be willing to

accept on intuition alone the truth of “if Paris is the capital of France, then Berlin is the capital of Germany.” Fortunately, our procedure can establish the truth of the first line as well, so we start our procedure with only the second line established by intuition alone.

9. An alternative approach might be to point out to the students that because the truth table for the biconditional has it that $P \equiv Q$ is true whenever P and Q have the same truth value, this could be describe in the following manner: “if P is true, then Q is also true, and if P is false, then Q is also false.” This could then be reformulated in symbols as $(P \supset Q) \cdot (\sim P \supset \sim Q)$. Our procedure will work using this alternative “definition” of the biconditional, but the negations add some extra complications, which we think are best avoided.

10. It should also be noted that all logical equivalence strategies, including Furman’s, have this same potential problem. They all require *some* intuitive understanding of the meaning of conditional talk in order to get off the ground in the first place.

11. This issue was pointed out to us by an anonymous referee, to whom we are grateful for the insight.

12. This insight also came from an anonymous referee.

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